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RESIDUAL AND SPECTRAL DECOMPOSITIONS OF SECOND-ORDER
DIFFERENTIAL OPERATORS ON GRAPHS

ABSTRACT

for the dissertation work of M.N.Konyrkulzhaeyva, for the degree of Doctor of
Philosophy (PhD) in the specialty "6D060100-Mathematics"

The relevance of research. From a mathematical point of view, graphs are interesting in that they are a good model for studying the properties of systems depending on the geometry and topology of space. Graphs are composed of zero-dimensional and one-dimensional manifolds and, in this sense, it is interesting how mixed dimension affects certain properties of mathematical objects defined on graphs.

On the practical side, primarily when calculating elastic structures, it is necessary to study the problems of articulation of singularly degenerate regions. Moreover, the design may consist of areas of different dimensions. An asymptotic analysis of the solutions of boundary value problems is required in domains that depend on sufficiently small dimensions of the domains and are contracted to the limit "skeleton", which can be represented as a union of sets of different dimensions. Often such objects are called stratified sets.

Purpose of the study. Due to the fact that the differential operators on the graphs are not "pure" differential, but are "hybrid" operators, therefore the goal of the dissertation is to transfer the mathematical apparatus that was created to study the spectral properties of purely differential operators to "hybrid" operators.

Provisions to be defended. A complete description of all possible self-adjoint restrictions of the maximum operator is given, the correct restrictions of the maximum operator are written out, and the formulas for their resolvents are found, the dependence of the resolvents on the lengths of the graph edges is established, and uniform resolvent convergence to the limit operator is investigated, spectral effects of the limit operator are presented.

The objectives of the study. Find the Green function for a differential operator on a graph with boundary conditions, construct the residue and spectral expansion of the Fourier series of the Green function of the second-order differential operator with boundary conditions. The next task is to investigate the behavior of the eigenvalues and corresponding eigenfunctions for two simplest graphs with small edges on the graphs.

Object and subject of research. Differential operators on graphs, as operators composed of scalar matrix and differential operators. To achieve this goal, the following tasks are investigated in a dissertation:

- correctly determine the maximum operator,
- derive the Lagrange formula for the maximum operator,
- build an adjoint operator,
- describe all kinds of self-adjoint restrictions of the maximum operator,
- describe the correct restrictions of the maximum operator and the derivation of the formula of their resolvent,
- find the dependence of the resolvent on the lengths of the edges of the graph and investigate the uniform resolvent convergence to the limit operator,
- indicate the spectral effects of the limit operator.

Research Methods. The dissertation uses well-tested methods of the classical theory of ordinary differential equations, linear differential operators, methods of functional analysis, the apparatus of the spectral theory of unbounded operators, methods of the theory of functions of a complex variable, and also methods of the theory of differential equations on graphs.

The scientific novelty of the study. The dependence of the resolvents on the lengths of the edges of the graph is established for the first time, and uniform resolvent convergence to the limit operator is shown. Significant spectral effects of the limit operator are indicated, which until now have not been paid due attention.

Validity and reliability. The validity and reliability of the scientific conclusions obtained in the thesis are confirmed by their consistent theoretical and mathematical justification, as well as experimental data, compared with the technological and production data available in open sources.

Theoretical and practical significance of the research. The work is theoretical. The results of the work can find further application in the development of the spectral theory of boundary value problems on graphs and in the study of problems arising in the theory of elasticity, theory of stability, etc.

Testing the results of the study. The obtained results on the topic of the work were presented at eight international conferences: the international scientific-practical conference “Contemporary Mathematics and its Applications”, 1 article, the international conference on function theory, dedicated to the 100th anniversary of A.F. Leontyev, 1 thesis, The International Scientific Conference “Weighted estimates of differential and integral operators and their applications”, 1 thesis, abstracts of reports of the VII international youth scientific-practical conference “Mathematical modeling of processes and systems”, 1 article, conference “Modern problems of mathematics, Mechanics and Informatics” dedicated to the 25th anniversary of Independence of the Republic of Kazakhstan”, Kazakhstan, 1 thesis,

conference “XIV International Scientific Conference of Students, Masters and Young Scientists “LOMONOSOV-2018”, 1 thesis, Abstracts book of the international scientific conference “Spectral Theory and related issues”, 1 article, international conference on mathematics: “An Istanbul Meeting for World Mathematicians”, 1 article. Based on the materials of the thesis, 14 published works were published, 3 of them in journals indexed in the Scopus database, 3 in journals recommended by the KKSON of the Ministry of Education and Science of the Republic of Kazakhstan, 8 - works in materials of international scientific conferences.

The reliability and validity of the scientific provisions, conclusions and results of the dissertation is confirmed by the publication of the results in journals that have a non-zero impact factor.

Assessment of the completeness of the objectives of the study. All the results obtained are new and are based on our own methods of solution. The existence of a residual decomposition of any functions from the domain of definition of the differential operator under consideration on a graph in a Fourier series in the eigenfunctions of a given boundary value problem.

Publications. Based on the results of the dissertation, 14 works were published: 6 journal articles (3 of them in indexed journals, 3 in journals recommended by the Committee for Control in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan), and 8 works in collections of international scientific conferences.

The results on the topic of the dissertation were published in the following papers:

Publications in rating scientific journals

1 Жапсарбаева Л.К., Кангужин Б.Е., Кобыркулжаева М.Н. Самосопряженные сужения максимального оператора на графе // Уфимский математический журнал. Т.9. выпуск 4, С.36-44, 2017.

2 Borisov D.I., Konyrkulzhayeva M.N. Perturbation of threshold of the essential spectrum of the Schrodinger operator on the simplest graph with a small edge // Journal of Mathematical Science, V.239, No.3, P.248-267, 2019.

3 Борисов Д.И., Кобыркулжаева М.Н. Простейшие графы с малыми ребрами: асимптотики резольвент и голоморфная зависимость спектра // Уфимский математический журнал. Т.11, №2, С.56-71, 2019.

Publications in journals recommended by the Committee for Control in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan

1 Жанузакова Д.Т., Кобыркулжаева М.Н., Таирова А.Б., Тулеухасым С.Т. Представление резольвенты дифференциального оператора на геометрическом графе // Вестник КазНПУ им.Абая. №1(61), С.61-68, 2018.

2 Жаруллаев Д.Б., Кангужин Б.Е., Кобыркулжаева М.Н. Функция Грина дифференциального оператора на графе-звезде с общими граничными условиями // Вестник КазНУ им. Аль-Фараби. №1(101), С.48-58, 2019.

3 Аймал Раса Г.Х., Аузерхан Г.С., Кобыркулжаева М.Н. Функция Грина задачи Дирихле дифференциального оператора на графе-звезде при m . Вестник КазНУ им. Аль-Фараби. №1(101), С.14-28, 2019.

Publications in materials of international conferences

1 Жапсарбаева Л.К., Кангужин Б.Е., Кобыркужаева М.Н. Сверточное представление резольвент дифференциального оператора на графе // Сборник докладов международной научно-практической конференции «Современная математика и ее приложения» Уфа, БашГУ. С.75-80, 21-23 мая, 2017.

2 Kanguzhin B.E., Zhapsarbayeva L.K., Konyrkulzhayeva M.N. Representation of the resolvent of second order differential operator on a graph // The International Scientific Conference «Weighted estimates of differential and integral operators and their applications» P.42, Astana, 3-5 may, 2017.

3 Жанай А.Ж., Кангужин Б.Е., Кобыркулжаева М.Н. Об одной задаче на собственные значения дифференциального уравнения третьего порядка // Тезисы докладов VII международной молодежной научно-практической конференции «Математическое моделирование процессов и систем», Уфа, С. 14-19, 2017.

4 Жанузакова Д.Т., Кобыркулжаева М.Н. Представление резольвенты дифференциального оператора на компактных графах // Конференция «Современные проблемы математики, механики и информатики», посвященной 25-летию Независимости Республики Казахстан", Казахстан, г. Караганда, С.28, 2016.

5 Кобыркулжаева М.Н. Функция Грина задачи Дирихле для оператора Штурма-Лиувилля // XIV Международная научная конференция студентов, магистрантов и молодых ученых «ЛОМОНОСОВ–2018», Казахстан, С.24-26, 2018.

6 Кобыркулжаева М.Н. Решение дифференциальных уравнений с первыми краевыми условиями на графе-звезде // Сборник тезисов международной научной конференции «Спектральная теория и смежные вопросы» Уфа, С.97-99, 2018.

7 Kanguzhin B.E., Konyrkulzhayeva M.N. Green's function of the

Dirichlet problem for the differential operator on a star-shaped graph // International conference on mathematics: «An Istanbul Meeting for World Mathematicians» Istanbul, Turkey, P.195-204, 2018.

8 Кангужин Б.Е., Коньркулжаева М.Н., Оператор, резольвенты которых имеют сверточное представление, и их спектральный анализ // Международную конференцию по теории функций, посвященный 100-летию А.Ф.Леонтьева. – Уфа, Институт математики с вычислительным центром УНЦ РАН, Башкирский государственный университет и Академия наук РБ. С.80, 15-мая, 2019.

The structure and scope of the dissertation. The dissertation consists of 143 pages, which include an introduction, three chapters with sections, a conclusion and a list of references.

The introduction of the dissertation contains a brief description of the current state of the research topic of the dissertation, the relevance and justification of the need for research. The first chapter first introduces the well-known concepts and statements concerning differential operators on arbitrary connected geometric graphs without loops. The maximum operator on the graph is determined by differential expressions on the edges, Kirchhoff conditions at the inner vertices of the graph. For the introduced maximal operator, an analog of the Lagrange formula is proved. For an arbitrary set of boundary conditions, an algorithm for constructing conjugate boundary forms is indicated. A complete description of all self-adjoint restrictions of the maximal operator is also given.

In the second chapter, we study the system of second-order differential equations, which is a model of oscillatory systems with a rod structure. The Green function of the Dirichlet problem for the differential operator on a star-shaped graph is derived. In this case, the standard Kirchhoff conditions at the inner vertex and the Dirichlet boundary conditions at the boundary vertices are used.

In the third chapter, we consider a graph consisting of two infinite edges and one small-length edges. On such a graph, the Schrödinger operator with piecewise constant potentials on infinite edges and a singular potential on a small edge is considered. The limiting boundary conditions are determined, the binomial asymptotics for the resolvents of the operators under consideration are obtained in the operator norm, and the residuals are estimated. The effect is also being studied.

The emergence of isolated eigenvalues from the edge of the essential spectrum. Effective, easily verified sufficient conditions for the existence and absence of such eigenvalues are established, and a holomorphic dependence of the arising eigenvalues from the edge of the essential spectrum is discovered. We also consider a stellar graph consisting of two infinite edges and one short-length edge. The length of the last edge is considered a small parameter in the problem. At the interior vertex of this graph, the interaction is specified, at the boundary vertex of

the small edge - the Dirichlet condition or the Neumann condition. The limit boundary conditions are obtained, the binomial asymptotics for the resolvents are obtained in the operator norm, and the residuals are estimated. The effect of the occurrence of isolated eigenvalues from the edge of the essential spectrum is studied. A holomorphic dependence of the arising eigenvalues from the edge of the essential spectrum was found.